DEVELOPMENT OF SIMULATION TOOL FOR FINDING OPTIMUM TILT ANGLES FOR SOLAR COLLECTORS

NAVEED UR REHMAN, MUBASHIR ALI SIDDQUI
MECHANICAL ENGINEERING DEPTARTMENT,
NED UNIVERSITY OF ENGINEERING AND TECHNOLOGY,
KARACHI, PAKISTAN
naveedurrehman@neduet.edu.pk

ABSTRACT

For efficient performance of photovoltaic (PV) panels and flat-plate solar collectors, one of the most important factors that should be considered is tilt angle. Tilt angles may be adjusted monthly, seasonally, half-yearly or they may be kept fixed throughout the year. Optimum values of tilt angle in any span of year are not the same everywhere and calculations become tedious if various sites are required to be evaluated. This paper discusses the development of web-based simulation tool with interactive map for finding optimum tilt angles at any location in the world. The isotropic sky model is assumed and the climatic data (monthly average daily global and diffused solar radiations on horizontal) are taken from NASA SSE. The results for different regions of the world, obtained through this tool are compared with reported values in literature, and high values of correlation coefficients have been found between the two.
1. INTRODUCTION

Tilt angle is the angle between horizontal and flat-plate solar collectors or PV panels (hereafter called “collectors”). For effective performance of collectors, there is an optimum tilt angle. Theoretically, the value of optimum tilt angle varies every day; while it also depends upon the available solar energy (global and it’s diffused/beam components on horizontal) as well as the location (latitude) where the collectors are placed.

The tilt angle should be adjusted each day to its optimum value for maximizing the collection of solar energy by collectors but this exercise may not be recommended if the penalty in output (in a long run) due to delayed adjustment is negligible. It is, therefore, suggested to adjust the tilt angle at least seasonally, if not monthly. Even if the half-yearly adjustments, compared with the seasonal one, are not harming the overall energy collection then they can be adopted. In many cases, the collectors are kept at fixed tilt angles. Although, the performance of a collector placed at a fixed tilt angle is less than those which are adjusted timely, advantages of former one include easy installation and low operating cost.

Researchers have reported that the optimum value of tilt angle for fixed-type collectors is nearly the latitude (\(\varphi\)), for example, \(\varphi + 20\) (Hottel, 1954), \(\varphi - 10\) (Heywood, 1971), \(\varphi + 10\) (Kern & Harris, 1975) and \(\varphi\) (El-kassaby, 1988) (Gopinathan, 1991) (Soulayman, 1991) (Morcos, 1994).

In literature, many researchers have recommended two values for optimum tilt angles, lower for summer and higher for winter, for example, \(\varphi \pm 20\) (Yellott, 1973), \(\varphi \pm 15\) (Lunde, 1980) (Garg, 1982) and \(\varphi \pm 8\) (Lewis, 1987).

During last two decades, researchers have made efforts for estimation of local optimum tilt angles for a specific location (city or country) using solar radiation data obtained through direct measurements, correlations or trusted resources.

Theoretical aspects of finding a tilt angle for solar flat-plate collectors in Cyprus (\(\varphi = 35.13^\circ N\)) were examined and recommendations were made on increasing solar energy collection by varying the tilt angle (Ibrahim, 1995). Hourly global solar energy data over 8 years (1986-1993), recorded by the meteorological office at the city of Guzelyurt, were used after converting it into monthly average daily radiations. For finding the optimum tilt angle, a computer simulation program was developed which first calculates monthly average daily total radiations at tilt angles varying from 0° to 90° and then suggests the value of tilt angle, corresponding to the maximum value of collected energy, as optimum. For Brunei Darussalam (\(\varphi = 4.53^\circ N\)), optimum values of tilt angle were determined on daily basis as well as for a specific period (Yakup & Malik, 2001).

The results for monthly-averaged optimum tilt angle were compared with equations derived by Nijegorodov et al. who presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60° south to 60° north (Nijegorodov, Devan, & Jain, 1997). Mathematical equations used by Nijegorodov et al. did not take into account the localized patterns of solar radiations falling over a particular location (region). Extensive work has
also been done for finding the optimal tilt angles in China. The study outlined a contour map of the fixed optimum tilt angle for collectors facing south, based on monthly horizontal radiations at 152 locations around the country (Tang & Wu, 2004). A study was also conducted for estimating monthly, seasonal, half-yearly and fixed optimum tilt angles for Izmir, Turkey (\( \phi = 38.42^\circ N \)) (Ulgen, 2006). Solar energy data were taken from meteorological station located at Solar Energy Institute Building in Ege University. A case study was also performed for optimizing tilt angle for Madinah, Saudi Arabia (\( \phi = 24.5^\circ N \)) (Benghanem, 2011) which utilized solar data available at National Renewable Energy Laboratory (NREL) website. Various efforts have been reported by Kumar, Thakur, Makade, & Shivhare (2011) for Khatkar Kalan (Punjab, India), Li & Lam (2007) for Hong Kong, and Bari (2000) and Bari (2001) for Malaysia.

Various simulation tools with interactive maps related to renewable energy are available, of which NREL-PVWatts Viewer is one of the popular one. It has an interactive map-based interface for providing access to multiple data sets which link directly to their corresponding PVWatts calculators. It can instantly determine the energy production and cost savings of grid-connected PV energy systems throughout the world. NREL-IMBY is another tool for estimating the amount of electricity that can be produced for household applications. It also provides generation analysis given the average system loads for a given location. Solar Calculator, powered by National Oceanic and Atmospheric Administration Earth System Research Laboratory (NOAA ESRL), allows user to pick the location of interest through an interactive map and helps finding the solar noon, sunrise and sunset times for any date. SunCalc is another similar but much more interactive application for determining the sun positions at sunrise, sunset and any specified time.

The main objective of this study is to develop a user-friendly simulation tool with interactive map interface for finding monthly, seasonal, half-yearly and fixed optimum tilt angles for any location in the world. The results of this study will be compared with the results, published by other researches, available in literature.

2. METHODOLOGY

Optimum tilt angle for solar collectors at any location is a function of latitude (\( \phi \)) and the quantity of solar radiations available at that location. If the above mentioned information is authentically provided, a simple method can be deployed for finding optimum tilt angles for any given span of the year. In the following sections, determination of geocode (latitude and longitude) of location, acquiring solar radiation data and the method for finding optimum tilt angle is discussed in details.

2.1 Map Interface

The map interface provides an easy and user-friendly interaction for picking up location of interest instead of entering the values of latitude and longitude. For the development of map interface in website, Google Maps JavaScript API V3 (About Google Maps) along with The Google
Geocoding API (About Google Geocoding API) was used. It is a free service, available for any web site that is free to consumers.

By default, the map object is set to display ‘Street Map’ at zoom level of ‘3’, so that country level information (names etc.) could be displayed. The map object is allowed to switch between “Satellite” and “Terrain” display modes. User may also zoom the map up to the desired level, provided that Google map supports that zoom level in the region of interest.

A marker (bubble) is also set to the default location geocode (latitude and longitude). The marker is draggable so that user can drag-drop it to the location of interest. On dropping the marker at any location, the geocode which is the input to the solar data acquiring system, is updated. Location’s real address (or name) will be obtained through Google reverse geocoding (address lookup).

Another feature added to the interface is “Find location”. User will be required to enter the location’s real address (or name). Resulting locations matching the query will be listed. On selecting any result, the marker on the map will dynamically move to the location along with updating the geocode. User may also enter the values of latitude and longitude.

2.2 Solar Data

NASA’s Surface meteorology and Solar Energy (SSE) data is used as the backend database for acquiring solar radiation data of the location of interest. NASA SSE data has been used in various research activities around the globe for such studies. Out of many other parameters, these data include long-term estimates of global solar radiations on horizontal surface in kWh/m²/day averaged over each month and trustworthy set of polynomials for estimating diffused horizontal radiations in terms of clearness index ($\overline{k}_T$), sunset hour angle (SSHA) and noon solar angle (NSA) from the horizon in degrees. These satellite and model-based products have also been shown to be accurate enough to provide reliable solar resource data over regions where surface measurements are sparse or nonexistent, and it also offers two unique features i.e. the data is global and, in general, contiguous in time (Surface meteorology and Solar Energy (SSE) Release 6.0 Methodology, 2012). This database has been used as benchmark by different researchers for comparing their derived correlations and empirical models for estimating the climatic conditions especially solar radiations (El-Sebaii, Al-Hazmi, Al-Ghamdi, & Yaghmour, 2010), (Islam, Alili, Kubo, & Ohadi, 2010) and (Zawilska & Brooks, 2011).

2.3 Model for Estimating Tilt Angle

The optimum tilt angle ($\beta_{opt}$) for a particular span of time, at any location, will be the one at which, if collector is tilted, will collect maximum solar energy compared with any other angle. The latitude ($\phi$) and the global solar radiation on horizontal ($H$) were obtained through sections 0 and 0. The following correlations provided in NASA SSE Methodology were used to find out the monthly diffused ($H_D$) component on horizontal (Surface meteorology and Solar Energy (SSE) Release 6.0 Methodology, 2012):

For latitudes between 0° and 45° North and South:
\[
\frac{H_D}{H} = 0.96268 - 1.45200\bar{K}_T + 0.27365\bar{K}_T^2 + 0.04279\bar{K}_T^3 + 0.000246(SSHA) + 0.001189(NHSA)
\]
For latitudes between 45° and 90° North and South:
If SSHA = 0° - 81.4°:
\[
\frac{H_D}{H} = 1.441 - 3.6839\bar{K}_T + 6.4927\bar{K}_T^2 - 4.417\bar{K}_T^3 + 0.0008(SSHA) + 0.008175(NHSA)
\]
If SSHA = 81.4° - 100°:
\[
\frac{H_D}{H} = 1.6821 - 2.5866\bar{K}_T + 2.3733\bar{K}_T^2 - 0.5294\bar{K}_T^3 - 0.00277(SSHA) - 0.004233(NHSA)
\]
If SSHA = 100° - 125°:
\[
\frac{H_D}{H} = 0.3498 + 3.8035\bar{K}_T - 11.765\bar{K}_T^2 + 9.1748\bar{K}_T^3 + 0.001575(SSHA) - 0.002837(NHSA)
\]
If SSHA = 125° - 150°:
\[
\frac{H_D}{H} = 1.6586 - 4.412\bar{K}_T + 5.8\bar{K}_T^2 - 3.1223\bar{K}_T^3 + 0.000144(SSHA) - 0.000829(NHSA)
\]
If SSHA = 150° - 180°:
\[
\frac{H_D}{H} = 0.6563 - 2.893\bar{K}_T + 4.594\bar{K}_T^2 - 3.23\bar{K}_T^3 + 0.004(SSHA) - 0.0023(NHSA)
\]
Where,
\[
\bar{K}_T = \frac{H}{H_0}
\]
\[
SSHA = \omega_s = \cos^{-1}(-\tan(\varphi - \beta) \tan \delta)
\]
For northern hemisphere: \( NHSA = 90 - |\varphi - \delta| \)
For southern hemisphere: \( NHSA = 90 - |\delta - \varphi| \)

The extraterrestrial solar radiation can be calculated using:
\[
H_0 = \frac{3600 \times 24}{\pi} G_{sc} \left( 1 + 0.033 \cos \frac{360n}{365} \right) \left( \cos \varphi \cos \delta \sin \omega_s + \frac{\pi}{180} \omega_s \sin \varphi \sin \delta \right)
\]
Where \( G_{sc} \) is solar constant (\( \approx \) 1367 \( W/m^2 \)) and \( n \) is the average day of the month (Klein, 1977) which can be found from the following table:

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>17</td>
<td>47</td>
<td>75</td>
<td>105</td>
<td>135</td>
<td>162</td>
<td>198</td>
<td>228</td>
<td>258</td>
<td>288</td>
<td>318</td>
<td>344</td>
</tr>
</tbody>
</table>

The declination angle (\( \delta \)) can be calculated as:
\[
\delta = 23.45 \sin \left( \frac{360 \times 284 + n}{365} \right)
\]
Using Liu & Jordan (1960), the beam component on tilted surface is determined using \( \bar{R}_b \), which is the ratio of the average beam radiation on tilted surface to that on horizontal surface for a month. Assuming isotropic sky with ground reflectance (\( \rho \approx 0.2 \)), total radiations on tilted surface (\( H_T \)) is determined using the following equation (Liu & Jordan, 1960):
\[ \bar{H}_T = (\bar{H} - \bar{H}_b)\bar{R}_b + \frac{\bar{H}_d}{2} (1 + \cos \beta) + \frac{\bar{H}_o}{2} (1 - \cos \beta) \]

For finding \( \beta_{opt} \) for a single month, the values of \( \beta \) in above equation must be looped from -90° to +90°. The \( \beta \) against which maximum \( \bar{H}_T \) is received should be \( \beta_{opt} \). For longer spans (e.g. season or half-year etc.), same procedure should be adopted. \( \beta_{opt} \) would be the value of \( \beta \) against which the maximum sum of all \( \bar{H}_T \) (of months in span) was received.

The function-based codes are written in PHP 5 to simulate above method. The input to the function was span of months, latitude and data for solar radiation on horizontal, of the location of interest. The function was returning optimum tilt angle and estimate of energy to be collected, for that span of given months.

3. RESULTS

Resulting tilt angles for different regions were gathered and compared with the results which were published by other researchers in literature. For the purpose, statistical test methods were used; namely Mean Bias Error (MBE), Root Mean Square Error (RMSE) and Correlation Coefficient \( (R^2) \). Summary of some of the comparisons is given in Tables 1 to 4.

4. CONCLUSION

From statistical analysis and graphical results, it is found that the tool is estimating tilt optimum angles with sufficient accuracy. For all the cases discussed, the correlation coefficient was found to be more than 0.9. Hence, the tool can reliably be used for quickly determining optimum tilt angles for any location in the world.

5. ONLINE ACCESSIBILITY

The tool is available online at http://www.naveedurrehman.com/demo/otilt2/

Nomenclature

- \( \bar{H}_T \) monthly average daily global radiations on tilted surface (W/m²/day)
- \( \bar{R}_T \) monthly clearness index
- \( \bar{H} \) monthly average daily global radiations on horizontal surface (W/m²/day)
- \( \bar{H}_d \) monthly average daily diffuse radiations on horizontal surface (W/m²/day)
- \( \bar{H}_b \) monthly average daily direct (beam) radiations on horizontal surface (W/m²/day)
- \( \bar{H}_o \) monthly average daily extraterrestrial radiations (W/m²/day)
- \( \bar{R}_b \) ratio of average beam radiation on the tilted surface to that on a horizontal surface

Greek Letters
\( \beta \) tilt angle (°)
\( \gamma \) surface azimuth angle (°)
\( \varnothing \) latitude of site of interest
\( \rho \) ground reflectivity (=0.2)

**Suffixes**

*opt* optimum
*max* maximum
6. REFERENCES


Monthly optimum tilt angles for Brunei, Darussalam

Published* Results

Monthly optimum tilt angles for Izmir, Turkey

Published* Results
Monthly optimum tilt angles for Beijing, China

Monthly optimum tilt angles for Khatkar Kalan, India
Table 1: Comparison of monthly tilt angles (in degrees) for solar collectors, facing south, reported for Brunei, Darussalam [4.5352°N, 114.7276°E] (Yakup & Malik, 2001)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Published*</td>
<td>28.3</td>
<td>20.5</td>
<td>5</td>
<td>-6.5</td>
<td>-19.3</td>
<td>-24.3</td>
<td>-24.3</td>
<td>-12</td>
<td>1.6</td>
<td>10.7</td>
<td>28.7</td>
<td>32.3</td>
</tr>
<tr>
<td>Results</td>
<td>32</td>
<td>22</td>
<td>7</td>
<td>-9</td>
<td>-21</td>
<td>-27</td>
<td>-24</td>
<td>-14</td>
<td>1</td>
<td>16</td>
<td>28</td>
<td>34</td>
</tr>
</tbody>
</table>

RESULT: MBE: 0.3583°  RMSE: 2.454°  R²: 0.996

Table 2: Comparison of monthly tilt angles (in degrees) for solar collectors, facing south, reported for Izmir, Turkey [38.4188°N, 27.1287°E] (Ulgen, 2006)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Published*</td>
<td>58</td>
<td>48</td>
<td>34</td>
<td>17</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>28</td>
<td>45</td>
<td>56</td>
<td>61</td>
</tr>
<tr>
<td>Results</td>
<td>62</td>
<td>52</td>
<td>39</td>
<td>21</td>
<td>6</td>
<td>-2</td>
<td>1</td>
<td>16</td>
<td>34</td>
<td>50</td>
<td>60</td>
<td>63</td>
</tr>
</tbody>
</table>

RESULT: MBE: 3.1666°  RMSE: 3.8514°  R²: 0.997
Table 3: Comparison of monthly tilt angles (in degrees) for solar collectors, facing south, reported for Beijing, China [39.9042°N, 116.4074°E] (Tang & W, 2004)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Published*</td>
<td>66</td>
<td>57</td>
<td>42</td>
<td>24</td>
<td>8</td>
<td>0</td>
<td>3</td>
<td>17</td>
<td>36</td>
<td>52</td>
<td>63</td>
<td>68</td>
</tr>
<tr>
<td>Results</td>
<td>67</td>
<td>58</td>
<td>42</td>
<td>23</td>
<td>7</td>
<td>-1</td>
<td>2</td>
<td>15</td>
<td>33</td>
<td>52</td>
<td>64</td>
<td>69</td>
</tr>
</tbody>
</table>

RESULT: \( \text{MBE: } -0.4166° \) \( \text{RMSE: } 1.3228° \) \( R^2: 0.999 \)

Table 4: Comparison of monthly tilt angles (in degrees) for solar collectors, facing south, reported for Khatkar Kalan (Punjab), India [31.1647°N, 76.0239°E] (Kumar, Thakur, Makade, & Shivhare, 2011)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Published*</td>
<td>60.5</td>
<td>50.5</td>
<td>36.5</td>
<td>18</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8.5</td>
<td>26</td>
<td>56.5</td>
<td>58.5</td>
<td>62.5</td>
</tr>
<tr>
<td>Results</td>
<td>58</td>
<td>48</td>
<td>34</td>
<td>16</td>
<td>-1</td>
<td>-7</td>
<td>-4</td>
<td>8</td>
<td>26</td>
<td>46</td>
<td>57</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 5: Comparison of monthly tilt angles (in degrees) for solar collectors, facing south, reported for Khatkar Kalan (Punjab), India [31.1647°N, 76.0239°E] (Kumar, Thakur, Makade, & Shivhare, 2011)

RESULT: \( \text{MBE: } -2.9583° \) \( \text{RMSE: } 4.1205° \) \( R^2: 0.993 \)